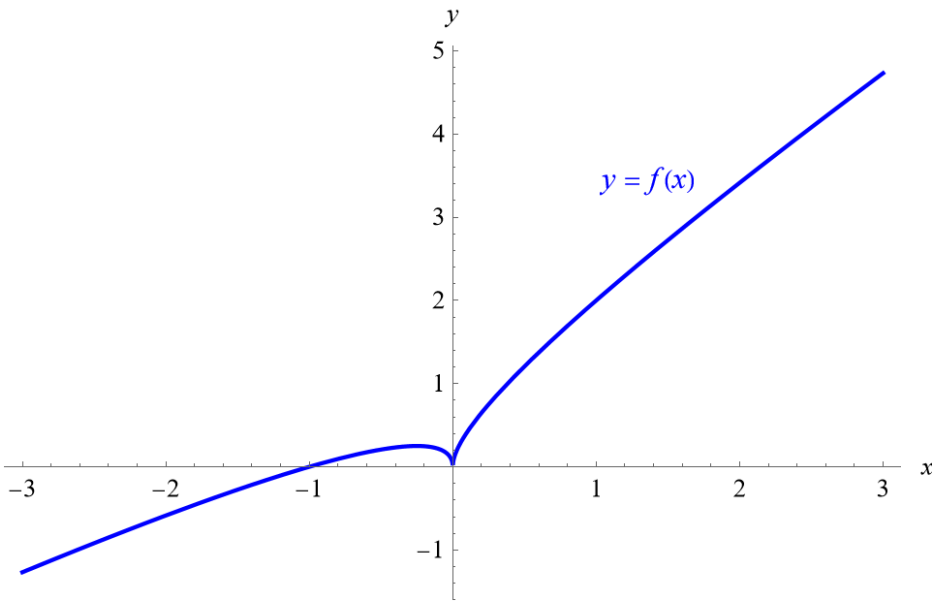
Exercise 45

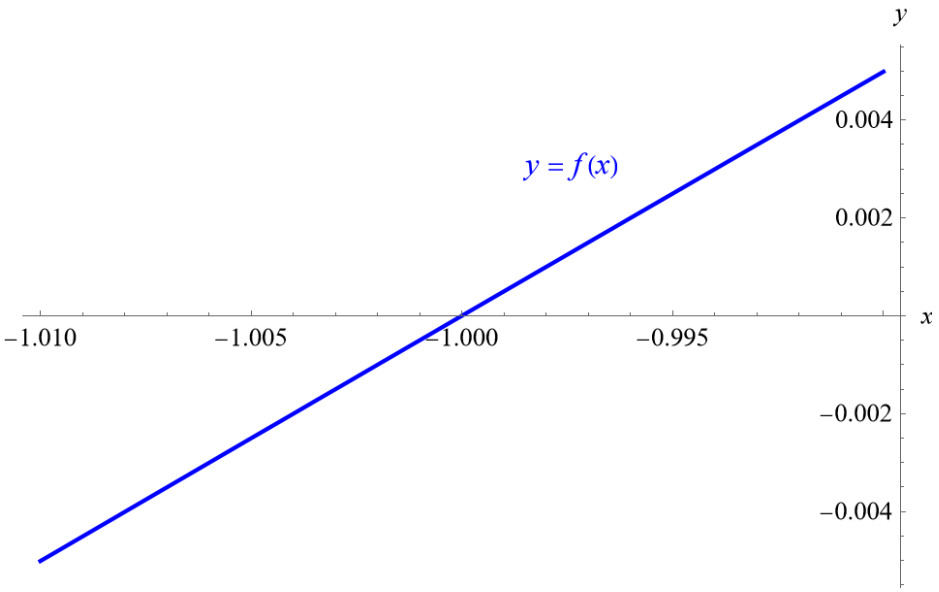
Graph the function $f(x) = x + \sqrt{|x|}$. Zoom in repeatedly, first toward the point $(-1, 0)$ and then toward the origin. What is different about the behavior of f in the vicinity of these two points? What do you conclude about the differentiability of f?

Solution

Below is a graph of $f(x)$ versus x.



Zoom in towards the point $(-1, 0)$. Notice that the graph is practically a straight line.



Now zoom in towards the point $(0, 0)$. Notice that no matter how much you zoom in, the graph never becomes a straight line.